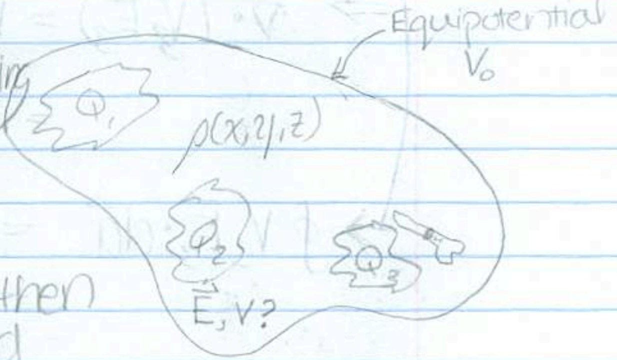
phys340 october 12, 2007

Second Uniqueness Theorem

In a volume surrounded by an equipotential, containing specified charge density $\rho(x,y,z)$ and containing individual conductors with total charges $Q_1, Q_2, Q_3, \ldots$, then the E-field within the closed boundary is uniquely determined.



Proof:

Assume two sol'ns: $\vec{E}_1, \vec{E}_2$

between conductors: $\vec{\nabla}\cdot\vec{E}_1 = \frac{\rho}{\varepsilon_0}$ $\quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

boundaries around conductors:

$$\oint_{\text{conductor } i} \vec{E}_1\cdot d\vec{a} = \frac{Q_i}{\varepsilon_0} \qquad \oint_{\text{conductor } i} \vec{E}_2\cdot d\vec{a} = \frac{Q_i}{\varepsilon_0}$$

outer boundary: $\oint_{\text{outer boundary}} \vec{E}_1\cdot d\vec{a} = \frac{Q_{tot}}{\varepsilon_0}$ $\quad \oint_{\text{outer boundary}} \vec{E}_2\cdot d\vec{a} = \frac{Q_{tot}}{\varepsilon_0}$

between conductors: $\vec{E}_3 = \vec{E}_2 - \vec{E}_1$

Ⓐ $\vec{\nabla}\cdot\vec{E}_3 = \vec{\nabla}\cdot(\vec{E}_2 - \vec{E}_1) = \vec{\nabla}\cdot\vec{E}_2 - \vec{\nabla}\cdot\vec{E}_1 = 0$

Ⓑ $\oint_{\substack{\text{outer boundary}\\ \text{boundaries of the conductors}}} \vec{E}_3\cdot d\vec{a} = 0$