15 Oct - 340 - 18

Laplace's Eq'n $\vec{\nabla}^2 V = \rho = 0$, how to find V?

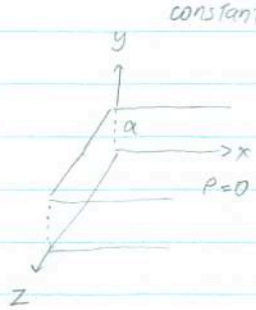
If the geometry of the Boundary Potentials can be described by separate variables, then so the sol'n $V(x,y,z) = X(x)\,Y(y)\,Z(z)$

$$\vec{\nabla}^2 V = YZ \frac{\partial^2}{\partial x^2} X(x) + XZ \frac{\partial^2}{\partial y^2} Y(y) + XY \frac{\partial^2}{\partial z^2} Z(z)$$

$$\frac{1}{V}\vec{\nabla}^2 V = \underbrace{\frac{1}{X}\frac{\partial^2}{\partial x^2} X(x)}_{\text{constant},} + \underbrace{\frac{1}{Y}\frac{\partial^2}{\partial y^2} Y(y)}_{C_2} + \underbrace{\frac{1}{Z}\frac{\partial^2}{\partial z^2} Z(z)}_{C_3} = 0$$

which $C_1 + C_2 + C_3 = 0$

ex



2-semiinfinite plane

① x-z plane $V=0$ at $y=0$, $y=a$

② $V=V_0$ when $x=0$

③ $V \to 0$ as $x \to \infty$

1. $\to Z(z) = 1$ 2. $\to \vec{\nabla}^2 V = C_1 + C_2 = 0$

$$\frac{\partial^2}{\partial x^2} X(x) = C_1 X(x) \quad \to \quad C_1 = K^2 \qquad C_2 = -C_1 = -K^2$$

$$X(x) = Ae^{kx} + Be^{-kx} \qquad Y(y) = C\sin(ky) + D\cos(ky)$$

Boundary ① $x \to \infty$ $V \to 0$ so $A = 0$

② $V=0$ at $y = 0$ n' a so $D = 0$ now make $B \times C = A$

now $Ae^{-kx}\sin(ka) = 0$ $k = \frac{n\pi}{a}$ $n \in \mathbb{Z}^+$ $\sum C_n \sin\frac{n\pi}{a}$

since $V(0, \cancel{0}, \omega, z) = V_0$ $\sum C_n \sin\frac{n\pi}{a} = V_0$

Hilroy