# Physics 257, Section One, Lab Five: Rotational Dynamics

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## 1 Data and Results

Apparatus no. 8.

The data in Table 1 are the specifications.

Table 2 are the measurements for Part 1.

Table 3 are the measurements for Part 1b.

Table 4 are the calculations for Part 1b.

We want to force the line to go through the origin, thus no calculations for $b$ are done in Table 5.

The data in Table 1 are the specifications; the moment of inertia of each disk calculated in Part 1.

Table 7 are the measurements for Part 2.

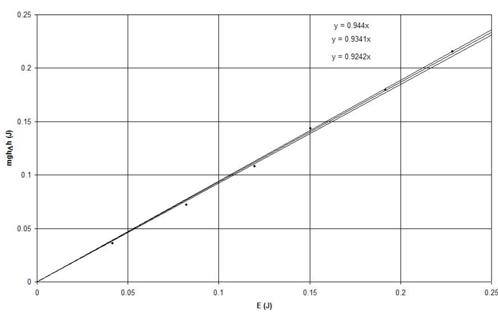


Figure 1: $mgh\Delta h$ *vs* $E$

Figure 1 demonstrates that energy is conserved.

## 2 Conclusion

### 2.1 Chris Payette

(Ed: I do'n't have Chris'.)

### 2.2 Neil Edelman

This experiment saw the investigation of a varied range of rotational collisions using air bearings to minimize friction. It was demonstrated that $\Gamma = I\alpha$, independent of the mass or rotation of the system, within a reasonable error based on the number of trials. $I$ was calculated experimentally using given values for the measurements of the disks and pulleys. The first measurement of velocity was not calculated where it did not fit with the other measurements because the initial time was not known except that it was at most 2 seconds; this is because the release was not coordinated with a measurement pulse. The error in System 3 was large, but reasonable for only 2 trials. In System 4, the same rotation was obtained as in System 1 leading to the conclusion that the air hose wasn't properly closed. Because of this, the analysis of the angular motion with the incorrect system configuration led to a meaningless value that did not (as it should not) agree with the conclusion. Next, the conservation of energy in rotational systems was confirmed by comparing $mg\Delta h : E$, with a value of $0.93[1]$. The discrepancy is easily accounted for especially by uncertainties in measurement of the position and time of the falling weight and also by friction. Finally, repeated measurements of angular momentum before and after multiple collisions between rotating discs of different masses were all consistently conservative within a reasonable error.

| **System** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **disk mass** ($kg$) | 1.35900 | 1.35900 | 1.35900 | 1.35900 |
| **disk radius** ($m$) | 0.06328 | 0.06328 | 0.06328 | 0.06328 |
| $I_{disk}$ ($kgm^2$) | 0.002720961533 | 0.002720961533 | 0.002720961533 | 0.002720961533 |
| **disk 2 mass (kg)** | | | | 1.34517 |
| **disk 2 radius (m)** | | | | 0.06316 |
| $I_{disk2}$ ($kgm^2$) | | | | 0.002683066397 |
| **pully mass** ($kg$) | 0.01000 | 0.01000 | 0.03600 | 0.01000 |
| **pully radius** ($m$) | 0.01250 | 0.01250 | 0.02500 | 0.01250 |
| $I_{pully}$ ($kgm^2$) | 0.000000780625 | 0.000000780625 | 0.000011250000 | 0.000000780625 |
| **hanging mass** ($kg$) | 0.02443 | 0.04987 | 0.02443 | 0.02443 |
| $I_{system}$ ($kgm^2$) | 0.002721742158 | 0.002721742158 | 0.002732211533 | 0.005404808555 |
| **average** $\alpha$ ($s^{-2}$) | 1.09272 | 2.187533333 | 2.67685 | 1.11156 |
| $I\alpha$ ($N$) | 0.002974102 | 0.005953902 | 0.00731372 | 0.006007769 |
| $\Gamma$ ($N$) | 0.002993441 | 0.006110638 | 0.005989277 | 0.002993441 |

Table 1: Part 1 Specifications

| | **time,** $t$ ($s$) | **digital reading** | $\omega$ ($s^{-1}$) | $\alpha$ ($s^{-2}$) |
|---|---|---|---|---|
| **System 1** | 0 | 0 | 0.0000 | |
| | 2 | 20 | 0.6280 | |
| | 4 | 93 | 2.9202 | 1.1461 |
| | 6 | 163 | 5.1182 | 1.0990 |
| | 8 | 232 | 7.2848 | 1.0833 |
| | 10 | 298 | 9.3572 | 1.0362 |
| | 12 | 368 | 11.5552 | 1.0990 |
| **System 2** | 0 | 0 | 0.0000 | |
| | 2 | 83 | 2.6062 | |
| | 4 | 227 | 7.1278 | 2.2608 |
| | 6 | 365 | 11.4610 | 2.1666 |
| | 8 | 501 | 15.7314 | 2.1352 |
| **System 3** | 0 | 0 | 0.0000 | |
| | 2 | 72 | 2.2608 | 1.1304 |
| | 4 | 341 | 10.7074 | 4.2233 |
| **System 4** | 0 | 0 | 0.0000 | |
| | 2 | 13 | 0.4082 | |
| | 4 | 87 | 2.7318 | 1.1618 |
| | 6 | 156 | 4.8984 | 1.0833 |
| | 8 | 227 | 7.1278 | 1.1147 |
| | 10 | 296 | 9.2944 | 1.0833 |
| | 12 | 367 | 11.5238 | 1.1147 |

Table 2: Data for Part 1

| **drop,** $h$ **(**$m$**)** | **time (**$s$**)** | $mg\Delta h$ **(**$J$**)** | $v$ **(**$ms^{-1}$**)** | $\omega$ **(**$s^{-1}$**)** | $\frac{1}{2}I\omega^2$ **(**$J$**)** | $\frac{1}{2}mv^2$ **(**$J$**)** | $E$ **(**$J$**)** | **diff** $E$ $mg\Delta h$ |
|---:|---:|---:|---:|---:|---:|---:|---:|---:|
| 0.15 | 4.34 | 0.035935663 | 0.069124424 | 5.532166784 | 0.041649282 | 5.83655E-05 | 0.041707647 | 14.87 |
| 0.30 | 6.18 | 0.071871325 | 0.097087379 | 7.770098331 | 0.082161813 | 0.000115138 | 0.082276951 | 13.50 |
| 0.45 | 7.68 | 0.107806988 | 0.1171875 | 9.378751501 | 0.119703553 | 0.000167747 | 0.119871301 | 10.60 |
| 0.60 | 9.14 | 0.143742651 | 0.131291028 | 10.50748527 | 0.150250029 | 0.000210554 | 0.150460583 | 4.57 |
| 0.75 | 10.12 | 0.179678314 | 0.148221344 | 11.86245249 | 0.191498756 | 0.000268358 | 0.191767114 | 6.51 |
| 0.90 | 11.12 | 0.215613976 | 0.161870504 | 12.95482222 | 0.22839148 | 0.000320058 | 0.228711538 | 5.90 |

Table 3: Energy Conservation Data

| $x = E$ **(**$J$**)** | $y = mg\Delta h$ **(**$J$**)** | $xy$ **(**$J^2$**)** | $x^2$ **(**$J^2$**)** | $(y - ax)^2$ **(**$J^2$**)** |
|---:|---:|---:|---:|---:|
| 0.041707647 | 0.035935663 | 0.001498792 | 0.001739528 | 0.000009146 |
| 0.082276951 | 0.071871325 | 0.005913354 | 0.006769497 | 0.000024851 |
| 0.119871301 | 0.107806988 | 0.012922964 | 0.014369129 | 0.000017364 |
| 0.150460583 | 0.143742651 | 0.021627603 | 0.022638387 | 0.000010205 |
| 0.191767114 | 0.179678314 | 0.034456392 | 0.036774626 | 0.000000297 |
| 0.228711538 | 0.215613976 | 0.049313404 | 0.052308968 | 0.000003882 |
| Sums: | | 0.125732508 | 0.134600134 | 0.000065746 |

Table 4: Energy Conservation Calculations

| | | |
|---|---|---|
| $a$ | 0.934118745 | (a represents $mg\Delta h : E$) |
| $\sigma_a$ | 0.009883868 | |

Table 5: Part 1b.

| | **mass,** $m$ **(**$kg$**)** | **radius,** $r$ **(**$m$**)** |
|---:|---:|---:|
| **low steel disk (**$kg$**)** | 1.34517 | 0.06316 |
| **high steel disk (**$kg$**)** | 1.35900 | 0.06328 |
| **aluminium disk (**$kg$**)** | 0.47027 | 0.06313 |

Table 6: Part 2 Specifications

| | **top disk digital reading** | $\omega$ **(**$s^{-1}$**)** | **ang.mom. (**$kgs^{-1}$**)** | **bottom disk digital reading** | $\omega$ **(**$s^{-1}$**)** | **ang.mom. (**$kgs^{-1}$**)** | **collided system digital reading** | $\omega$ **(**$s^{-1}$**)** | **ang.mom. (**$kgs^{-1}$**)** |
|---|---:|---:|---:|---:|---:|---:|---:|---:|---:|
| steel top | 366 | 11.4924 | 15.6182 | | | | 181 | 5.6834 | 15.3689 |
| steel base | 389 | 12.2146 | 16.5996 | | | | 186 | 5.8404 | 15.7934 |
| | 350 | 10.9900 | 14.9354 | | | | 177 | 5.5578 | 15.0292 |
| | 303 | 9.5142 | 12.9298 | | | | 142 | 4.4588 | 12.0574 |
| | 321 | 10.0794 | 13.6979 | | | | 161 | 5.0554 | 13.6707 |
| aluminium top | 378 | 11.8692 | 5.5817 | | | | 95 | 2.9830 | 5.4155 |
| steel base | 354 | 11.1156 | 5.2273 | | | | 92 | 2.8888 | 5.2444 |
| | 338 | 10.6132 | 4.9911 | | | | 85 | 2.6690 | 4.8454 |
| | 307 | 9.6398 | 4.5333 | | | | 77 | 2.4178 | 4.3894 |
| | 321 | 10.0794 | 4.7400 | | | | 81 | 2.5434 | 4.6174 |
| steel top | 583 | 18.3062 | 24.8781 | -728 | -22.8592 | -30.7495 | -70 | -2.1980 | -5.9438 |
| steel base | 562 | 17.6468 | 23.9820 | -494 | -15.5116 | -20.8657 | 22 | 0.6908 | 1.8680 |
| | 584 | 18.3376 | 24.9208 | -475 | -14.9150 | -20.0632 | 31 | 0.9734 | 2.6322 |
| | 231 | 7.2534 | 9.8574 | -45 | -1.4130 | -1.9007 | 89 | 2.7946 | 7.5571 |
| | 82 | 2.5748 | 3.4992 | -494 | -15.5116 | -20.8657 | -199 | -6.2486 | -16.8973 |
| aluminium top | 425 | 13.3450 | 6.2758 | -294 | -9.2316 | -12.4181 | -112 | -3.5168 | -6.3845 |
| steel base | 343 | 10.7702 | 5.0649 | -246 | -7.7244 | -10.3906 | -97 | -3.0458 | -5.5295 |
| | 249 | 7.8186 | 3.6769 | -47 | -1.4758 | -1.9852 | 22 | 0.6908 | 1.2541 |
| | 386 | 12.1204 | 5.6999 | -140 | -4.3960 | -5.9134 | -8 | -0.2512 | -0.4560 |

Table 7: Part 2 Data

# A Sample Calculations and Error Analysis

## A.1 Part 1a

To calculate the angular velocity:

$$\begin{aligned}\omega &= \text{DigitalReading} \cdot 0.0314s^{-1}\\ &= 20 \cdot 0.0314s^{-1}\\ &= 0.628s^{-1}\end{aligned}$$

To calculate angular acceleration:

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t}\\ &= \frac{(2.9202s^{-1} - 0.628s^{-1})}{(4s - 2s)}\\ &= 1.1461s^{-2}\end{aligned}$$

To calculate moment of inertia of a disk or a pulley:

$$\begin{aligned}I &= \frac{M \cdot r^2}{2}\\ &= \frac{1.35900kg \cdot (0.06328m)^2}{2}\\ &= 0.002720962kgm^2\end{aligned}$$

To calculate the moment of inertia of a system:

$$\begin{aligned}I_{system} &= I_{disk} + I_{pully}\\ &= 0.002720962kgm^2 + 0.000000780625kgm^2\\ &= 0.002721742kgm^2\end{aligned}$$

To calculate average angular acceleration of a system:

$$\begin{aligned}\bar{\alpha} &= \frac{\sum_{i=1}^{N} \alpha_i}{N}\\ &= \frac{(1.1461s^{-2} + 1.099s^{-2} + 1.0833s^{-2} + 1.0362s^{-2} + 1.099s^{-2})}{5}\\ &= 1.09272s^{-2}\end{aligned}$$

To calculate the torque of system (method 1:)

$$\begin{aligned}\Gamma &= I_{system} \cdot \bar{\alpha}\\ &= 0.002721742kgm^2 \cdot 1.09272s^{-2}\\ &= 0.002974102N\end{aligned}$$

To calculate the torque of a system (method 2:)

$$\begin{aligned}\Gamma &= m \cdot g \cdot r \\ &= 0.02443kg \cdot 9.80643ms^{-2} \cdot 0.01250m \\ &= 0.002993441N\end{aligned}$$

## A.2 Part 1b

To calculate potential energy lost:

$$\begin{aligned}U &= m \cdot g \cdot h \\ &= 0.02443kg \cdot 9.80643ms^{-2} \cdot 0.15m \\ &= 0.035935663J\end{aligned}$$

To calculate the acceleration of the falling mass:

$$\begin{aligned}d &= V_o \cdot t + \frac{a \cdot t^2}{2} \\ a &= \frac{2 \cdot d}{t^2} \\ &= \frac{2 \cdot 0.15m}{(4.34s)^2} \\ &= 0.0159273ms^{-2}\end{aligned}$$

To calculate the velocity of the falling mass:

$$\begin{aligned}v &= \sqrt{2a \cdot \Delta h} \\ &= \sqrt{2 \cdot 0.0159273ms^{-2} \cdot 0.15m} \\ &= 0.069124424ms^{-2}\end{aligned}$$

To calculate the angular velocity of the disk:

$$\begin{aligned}\omega &= \frac{v}{r} \\ &= \frac{0.069124424ms^{-1}}{0.02499m} \\ &= 5.532166784s^{-1}\end{aligned}$$

To calculate rotational kinetic energy:

$$\begin{aligned}K_{rotational} &= \frac{I \cdot \omega^2}{2} \\ &= \frac{(0.002721742kgm^2)}{(5.532166784s^{-1})^2 \cdot 2} \\ &= 0.041649282J\end{aligned}$$

To calculate the kinetic energy due to the falling mass:

$$\begin{aligned} K_{falling} &= \frac{m \cdot v^2}{2} \\ &= \frac{0.02443kg \cdot (0.069124424ms^{-1})^2}{2} \\ &= 0.0000583655J \end{aligned}$$

To calculate the total kinetic energy:

$$\begin{aligned} E &= K_{rotational} + K_{falling} \\ &= \frac{i \cdot \omega^2}{2} + \frac{m \cdot v^2}{2} \\ &= 0.041649282J + 0.0000583655J \\ &= 0.041708J \end{aligned}$$

To calculate percent difference between $E$ and $mg\Delta h$:

$$\begin{aligned} P &= \frac{|E - mg\Delta h| \cdot 2}{E + mg\Delta h} \cdot 100\% \\ &= \frac{|0.041708J - 0.035935663J| \cdot 2}{0.041708J + 0.035935663J} \cdot 100\% \\ &= 14.87\% \end{aligned}$$

Finding the slope $a$:

$$\begin{aligned} a &= \frac{\sum xy}{\sum x^2} \\ &= \frac{0.125732508J^2}{0.134600135J^2} \\ &= 0.934118745 \end{aligned}$$

Finding the error on $a$:

$$\begin{aligned} \sigma_a &= \sqrt{\frac{\frac{1}{N-1}\sum_{i=1}^{N}(y - ax)^2}{\sum x^2}} \\ &= \sqrt{\frac{\frac{1}{5}(6.5746 \times 10^{-5}J^2)}{0.134600134J^2}} \\ &= 0.009883868 \end{aligned}$$

## A.3 Part 2

Calculating angular momentum:

$$\begin{aligned} L &= m \cdot \omega \\ &= (1.34517kg + 1.35900kg) \cdot (-2.1980s^{-1}) \\ &= -5.9438kgs^{-1} \end{aligned}$$