198-257A, Lab Four: The Spectrometer

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1 Data and Results

Apparatus no 8, with Hg lamp no 9.

2 Conclusion

2.1 Chris Payette

(Ed: I do'n't have Chris'.)

2.2 Neil Edelman

Using the known wavelengths of spectral lines for sodium, a diffraction grating was calibrated and used to measure the wavelengths of four selected spectral lines for mercury. The measured wavelengths were $4348[3]\mathring{A}$, $5446[3]\mathring{A}$, $5761[3]\mathring{A}$, and $5780[3]\mathring{A}$. These values are off by about 0.2% from the corresponding accepted values for the same colours, the difference did not fall within the calculated uncertainty. Worth noting was that every value was about $10\mathring{A}$ lower than the accepted value, suggesting a systematic error was introduced – most likely from an error in the measurement of the central angle. If the values were adjusted by this amount, the accepted values would fall within the uncertainty.

n	**angle**	**angle to n_0 ($\pm 30'$)**	d	σ_d
-4	$66°24'$	$44°26'$	33670.491	71.5
-3	$79°12'$	$31°38'$	33707.587	152
-2	$90°21'$	$20°29'$	33680.550	376
-1	$100°45'$	$10°05'$	33658.834	1580
1	$120°57'$	$10°07'$	33549.081	1570
2	$131°18'$	$20°28'$	33706.799	376
3	$142°27'$	$31°37'$	33723.513	152
4	$155°15'$	$44°25'$	33680.486	71.5
0	110°50’	weighted average:	33682.931	45.0

Table 1: Centre Lines in Na Spectrum

n	**angle**	**angle to n_0 ($\pm 30'$)**	λ **($\mathring{A}$)**	$\sigma_{\mathring{A}}$ **($\mathring{A}$)**
royal blue				
-3	$88°02'$	$22°48'$	4350.8	259
-2	$95°54'$	$14°56'$	4339.8	271
-1	$103°24'$	$07°26'$	4357.5	278
1	$118°15'$	$07°25'$	4347.8	278
2	$125°44'$	$14°54'$	4330.4	271
3	$133°41'$	$22°51'$	4359.8	260
		weighted average:	4347.823	109.8891
green				
-3	$81°47'$	$29°03'$	5451.7	246
-2	$91°59'$	$18°51'$	5441.2	266
-1	$101°29'$	$09°21'$	5472.1	277
1	$120°03'$	$09°13'$	5394.8	277
2	$129°44'$	$18°54'$	5455.1	266
3	$139°55'$	$29°05'$	5457.4	246
		weighted average:	5446.186	106.9772
yellow I				
-3	$79°58'$	$30°52'$	5760.1	242
-2	$90°49'$	$20°01'$	5764.6	264
-1	$100°10'$	$09°50'$	5752.3	276
1	$120°40'$	$09°50'$	5752.3	276
2	$130°52'$	$20°02'$	5764.2	264
3	$141°45'$	$30°55'$	5768.5	242
		weighted average:	5760.796	105.9393
yellow II				
-3	$79°50'$	$31°00'$	5782.5	241
-2	$90°46'$	$20°01'$	5764.6	264
-1	$100°57'$	$09°53'$	5781.3	276
1	$120°43'$	$09°53'$	5781.3	276
2	$130°56'$	$20°06'$	5787.6	264
3	$141°50'$	$31°00'$	5782.5	241
		weighted average:	5780.093	105.7708

Table 2: Selected Lines from Hg Spectrum

A Sample Calculations and Error Analysis

A.1 Converting DMS to DD for trigonometric functions

$$D^\circ M' S'' = \left(D + \frac{M}{60} + \frac{S}{3600}\right)^\circ$$
$$44^\circ 26' = 44.43^\circ$$

A.2 Calculating angular difference

$$\theta_n = |\phi_n - \phi_0|$$
$$\theta_{-4} = |66^\circ 24' - 110^\circ 50'|$$
$$= 44^\circ 26'$$

A.3 Calculating wavelength of the centre of the two Na lines

$$\lambda = \frac{\lambda_1 + \lambda_2}{2}$$
$$= \frac{5890\mathring{A} + 5896\mathring{A}}{2}$$
$$= 5893\mathring{A}$$

A.4 Calculating the number of lines

$$d_n = \frac{n\lambda}{\sin\theta_n}$$
$$d_{-4} = \frac{4(5893\mathring{A})}{\sin(44^\circ 26')}$$
$$= 33670.491$$

where the error on this value is:

$$\sigma_{dn} = \sqrt{\sigma_\theta^2 \left(-\lambda(\sin^{-2}\theta)(\cos\theta)\right)^2}$$
$$\sigma_{d-4} = \sqrt{(30')^2 \left(-(-5892\mathring{A})(\sin^{-2} 44^\circ 26')(\cos 44^\circ 26')\right)^2}$$
$$= 71.5$$

error on the weighted average of d:

$$\sigma_{dn} = \sqrt{\frac{1}{\sum_{n=1}^{N} \frac{1}{\sigma_{dn}^2}}}$$
$$= \sqrt{\frac{1}{\frac{1}{71.5^2} + ... + \frac{1}{71.5^2}}}$$
$$= 45.0$$

weighted average of d:

$$
\begin{aligned}
d &= \sigma_d^2 \sum_{n=1}^{N} \frac{d_n}{\sigma_{dn}^2} \\
d &= \left(\frac{1}{\frac{1}{71.5^2} + ... + \frac{1}{71.5^2}} \right)^2 \left(\frac{33670.491}{71.5^2} + ... + \frac{33680.486}{71.5^2} \right) \\
&= 33682.931
\end{aligned}
$$

A.5 Finding the wavelength of one trial for one spectral line

$$
\begin{aligned}
\lambda_{cn} &= \frac{\sin\theta_{cn} d}{n} \\
\lambda_{b-3} &= \frac{\sin(22^\circ 48')33682.931}{4} \\
&= 4350.8\mathring{A}
\end{aligned}
$$

where the error on this value is:

$$
\begin{aligned}
\sigma_{\lambda cn} &= sqrt(\sin^2\theta_{cn})\sigma_d^2 + (d^2\cos^2\theta_{cn})\sigma_{\theta cn}^2 \\
\sigma_{\lambda b-3} &= sqrt(\sin^2(22^\circ 48'))(45.0)^2 + ((33682.931)^2\cos^2(22^\circ 48'))(30')^2 \\
&= 259\mathring{A}
\end{aligned}
$$

A.6 Finding the error on the weighted average wavelength for one spectral line

$$
\begin{aligned}
\sigma_{\lambda c} &= \sqrt{\frac{1}{\sum_{n=1}^{N} \frac{1}{\sigma_{\lambda cn}^2}}} \\
&= \sqrt{\frac{1}{\frac{1}{259^2} + ... + \frac{1}{260^2}}}\mathring{A} \\
&= 109.8891\mathring{A}
\end{aligned}
$$

A.7 Finding the weighted average wavelength for one spectral line

$$
\begin{aligned}
d &= \sigma_{\lambda c}^2 \sum_{n=1}^{N} \frac{\lambda_{cn}}{\sigma_{\lambda cn}^2} \\
&= \left(\frac{1}{\frac{1}{259^2} + ... + \frac{1}{260^2}} \right)^2 \left(\frac{4350.8}{259^2} + ... + \frac{4359.8}{260^2} \right)\mathring{A} \\
&= 4347.823\mathring{A}
\end{aligned}
$$

A.8 Calculating the percent difference of the experimental λ to the accepted value

$$\begin{aligned} pd &= \frac{|exp - acc|}{acc} \cdot 100\% \\ &= \frac{\left|4347\mathring{A} - 4358\mathring{A}\right|}{4358\mathring{A}} \cdot 100\% \\ &= 0.25\% \end{aligned}$$