198-257A, Lab Three: Kater's Pendulum

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1 Data and Results

See Table 1 and Table 2.

The value obtained for g is $9.7920[8]ms^{-2}$, with a percentage error of 0.15% from the accepted value.

2 Conclusion

2.1 Chris Payette

(Ed: I do'n't have Chris'.)

2.2 Neil Edelman

A high-precision pendulum setup was used to measure g with a value of $9.7920[8]ms^{-2}$. Although this value is within 0.15% of the accepted value of $9.806431ms^{-2}$, it does not fall within the calculated accuracy of the experimental value. At the approximately six degree angle of oscillation used, the systematic error introduced by the use of a small angle approximation (around 0.05%) is about one third of the difference between the two values. Another significant source of systematic error was the assumption that T_1 and T_2 were precisely equal. The measured averages were within 0.001% of each other, but – comparing the difference with the error on the averages using only ten trials – there is about a 10% uncertainty on this 0.001% difference.

knife to knife, $h_1 + h_2$ **(**$\pm 0.01mm$**)**	966.02	
$h_1(\pm 2mm)$	308	(never used)
$h_2(\pm 2mm)$	658	(never used)

Table 1: Our equipment.

knife to small mass ($\pm 0.01mm$**)**	**trial**	$4T_1(\pm 0.001s)$	$4T_2(\pm 0.001s)$
7.20	1	7.892	7.897
	2	7.892	7.896
	3	7.891	7.896
	average T ($\pm 0.00014s$)	1.97292	1.97408
5.00	1	7.904	7.914
	2	7.905	7.915
	3	7.905	7.914
	average T ($\pm 0.00014s$)	1.97617	1.97858
9.00	1	7.891	7.882
	2	7.891	7.882
	3	7.891	7.882
	average T ($\pm 0.00014s$)	1.97275	1.97050
6.00	1	7.900	7.908
	2	7.900	7.908
	3	7.900	7.908
	average T ($\pm 0.00014s$)	1.97500	1.97700
8.00	1	7.893	7.890
	2	7.894	7.889
	3	7.893	7.890
	average T ($\pm 0.00014s$)	1.97333	1.97242
7.35	1	7.894	7.894
	2	7.894	7.893
	3	7.894	7.893
	average T ($\pm 0.00014s$)	1.97350	1.97333
7.30	1	7.894	7.896
	2	7.895	7.896
	3	7.895	7.895
	average T ($\pm 0.00014s$)	1.97367	1.97392
7.33	1	7.894	7.894
	2	7.894	7.894
	3	7.894	7.894
	4	7.894	7.894
	5	7.894	7.893
	6	7.894	7.895
	7	7.894	7.895
	8	7.895	7.894
	9	7.894	7.894
	10	7.894	7.895
	average T ($\pm 0.00008s$)	1.97353	1.97355

Table 2: Note: the two average values for T are close enough relative to their error that we will consider them to be equal at their average, $1.9354s$.

A Sample Calculations and Error Analysis

A.1 Calculating the average period for a trial

$$\begin{aligned}\bar{T} &= \frac{1}{N}\sum_{n=1} N\frac{4T_n}{4} \\ &= \frac{1}{3}\left(\frac{7.892s}{4}+\frac{7.892s}{4}+\frac{7.891s}{4}\right) \\ &= 1.9729s\end{aligned}$$

where the error on this value is:

$$\begin{aligned}\sigma_{\bar{T}} &= \frac{1}{4}\sqrt{\frac{1}{\sum_{n=1}^{N}\frac{1}{\sigma_{T_n}^2}}} \\ &= \frac{1}{4}\sqrt{\frac{1}{\left(\frac{3}{(0.001s)^2}\right)}} \\ &= 0.00014s\end{aligned}$$

calculating g:

$$\begin{aligned}g &= \frac{4\pi^2(h_1+h_2)}{T^2} \\ g &= \frac{4\pi^2(966.02mm)}{(1.9735s)^2} \\ g &= 9792.004mms^{-2}\end{aligned}$$

where the error on this value is:

$$\begin{aligned}\sigma_g &= \sqrt{\left(\left(\frac{\partial g}{\partial(h_1+h_2)}\right)^2\Bigg|_{h_1+h_2}\sigma_{h1+h2}^2\right)+\left(\left(\frac{\partial g}{\partial T}\right)^2\Bigg|_{\bar{T}}\sigma_{\bar{T}}^2\right)} \\ &= \sqrt{\left(\left(\frac{4\pi^2}{(1.9735s)^2}\right)^2(0.01mm)^2\right)+\left(\left(\frac{-8\pi^2(966.02mm)}{(1.9735s)^3}\right)^2(0.00008mm)^2\right)} \\ &= 0.8002mms^{-2}\end{aligned}$$

percentage difference:

$$\begin{aligned}p &= \frac{|g_{acc}-g_{exp}|}{\frac{1}{2}(g_{acc}+g_{exp})}\cdot 100\% \\ &= \frac{\left|(9.806431ms^{-2})-(9.7920ms^{-2})\right|}{\frac{1}{2}((9.806431ms^{-2})+(9.7920ms^{-2}))}\cdot 100\% \\ &= 0.15\%\end{aligned}$$