198-257A, Lab One: Statistics and Measurements

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1 Data and Results

For histograms refer to included sketches.

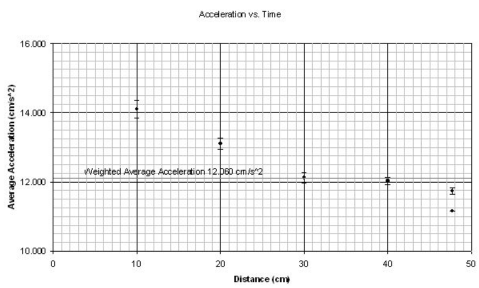

Figure 1: Acceleration *vs* Time

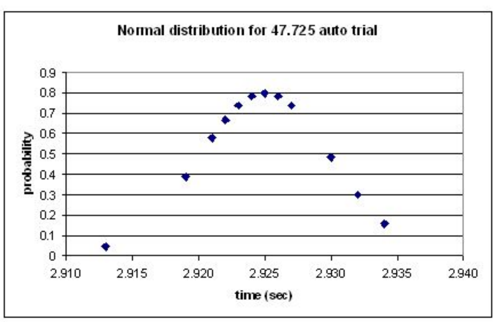

Figure 2: Probability *vs* Time

2 Conclusion

2.1 Chris Payette

Our result for the average acceleration of the cylinder was $12.060 \pm 0.003 cms^{-2}$. Judging by the small error associated with the value, it appears as though our results were very precise. However, the comparison of the worst data set to the weighted average revealed that there was in fact systematic error in our experiment. Since no effort has been made to estimate this error, the conclusion is that the results are probably not very accurate. To improve the accuracy of the experiment, one could ideally create a series of switches on the incline so that all the measurements for the time would be recorded by the computer. This would eliminate any systematic error that may have been associated with the way that a human stopped the clock. I feel this would improve the results, because the computer's trial at $47.725cm$ had the smallest standard deviation and also had the histogram that most resembled a Gaussian distribution. To increase the precision of the experiment we could also have performed more trials at each of the distances.

2.2 Neil Edelman

This experiment saw a cylinder rolling down various distances along an inclined plane at a constant inclination. It was shown that the acceleration was constant, independent of time or distance, and was computed to be $12.060 \pm .006 ms^{-2}$ for the apparatus used (no. 5.) The errors in measurement were analyzed and it was found that the measurements fell into a Gaussian distribution with greater precision for measurements of longer times and with the automatic timer.

Distance (cm)	47.725	47.725	40	30	20	10
Measurement Type	Auto	Manual	Manual	Manual	Manual	Manual
Time (s) for Trial No. 1	2.913	2.857	2.558	2.148	1.811	1.192
2	2.919	2.869	2.569	2.294	1.772	1.158
3	2.924	2.912	2.650	2.315	1.760	1.168
4	2.922	2.852	2.631	2.350	1.741	1.208
5	2.924	2.925	2.635	2.183	1.761	1.218
6	2.921	2.832	2.599	2.283	1.734	1.200
7	2.924	2.918	2.527	2.185	1.738	1.186
8	2.925	2.906	2.653	2.238	1.819	1.151
9	2.926	2.913	2.635	2.223	1.674	1.287
10	2.926	2.760	2.605	2.134	1.627	1.185
11	2.926	2.844	2.643	2.134	1.741	1.135
12	2.923	2.848	2.547	2.174	1.742	1.165
13	2.927	2.825	2.498	2.252	1.815	1.328
14	2.925	2.890	2.532	2.221	1.734	1.172
15	2.930	2.789	2.603	2.316	1.744	1.184
16	2.926	2.802	2.609	2.217	1.761	1.135
17	2.932	2.849	2.593	2.218	1.804	1.162
18	2.932	2.770	2.496	2.178	1.734	1.167
19	2.930	2.852	2.503	2.213	1.736	1.171
20	2.934	2.847	2.501	2.227	1.693	1.242
Average Time (s)	2.925	2.853	2.579	2.225	1.747	1.191
Std. Deviation (s)	0.005	0.048	0.055	0.062	0.047	0.048
Acceleration (cms^{-2})	11.153	11.727	12.025	12.118	13.105	14.107
Std. Deviation (cms^{-2})	0.008	0.089	0.115	0.150	0.157	0.255
Sq. Av. Std. Dev. (cm^2s^{-4})	0.00349					
Weighted Av. Accel. (cms^{-2})	12.060					

Table 1: Results from Apparatus no. 5

A Sample Calculations and Error Analysis

For $x = 47.725cm$ automatic:

A.1 Average time

$$\bar{t_{47A}} = \frac{\left(\sum_{n-1}^{N} T_n\right)}{N}$$
$$= \frac{(2.913s + 2.919s + 2.924s + 2.929s + ... + 2.934s)}{20}$$
$$= 2.925s$$

A.2 Std deviation on average time

$$\sigma_{t47A} = \sqrt{\frac{1}{N+1}\sum_{n-1}^{N}(t_i - \bar{t})^2}$$
$$= \sqrt{\frac{1}{19}\left((2.913s - 2.925s)^2 + (2.919s - 2.925s)^2 + ... + (2.934s - 2.925s)^2\right)}$$
$$= 0.005s$$

A.3 Acceleration

$$a_{47A} = \frac{2x}{\bar{t}^2}$$
$$= \frac{2(47.725cm)}{(2.925s)^2}$$
$$= 11.153cms^{-2}$$

A.4 Std dev on acceleration

$$\sigma_{\bar{47}A} = \sqrt{\frac{da}{dt}^2 \cdot \frac{\sigma_t}{\sqrt{20}}^2}$$
$$= \sqrt{\frac{-4x}{\bar{t}^3}^2 \cdot \frac{\sigma_t}{\sqrt{20}}^2}$$
$$= \sqrt{\frac{-4(47.725cm)}{(2.925s)^3}^2 \cdot \frac{(0.005s)}{\sqrt{20}}^2}$$
$$= 0.008cms^{-2}$$

A.5 Square of average std dev

$$
\begin{aligned}
\sigma^2 &= \frac{1}{\frac{1}{\bar{\sigma_{47}}^2} + \frac{1}{\bar{\sigma_{40}}^2} + \frac{1}{\bar{\sigma_{30}}^2} + \frac{1}{\bar{\sigma_{20}}^2}} \\
&= \frac{1}{\frac{1}{(0.089cms^{-2})^2} + \frac{1}{(0.115cms^{-2})^2} + \frac{1}{(0.150cms^{-2})^2} + \frac{1}{(0.157cms^{-2})^2}} \\
&= 0.00349cm^2s^{-4}
\end{aligned}
$$

A.6 Weighted average acceleration

$$
\begin{aligned}
\bar{a} &= \left(\frac{a_{47}}{\bar{\sigma_{47}}^2} + \frac{a_{40}}{\bar{\sigma_{47}}^2} + \frac{a_{30}}{\bar{\sigma_{47}}^2} + \frac{a_{20}}{\bar{\sigma_{47}}^2} \right) \cdot \sigma^2 \\
&= \left(\frac{11.727}{0.089^2} + \frac{12.025}{0.115^2} + \frac{12.118}{0.150^2} + \frac{13.105}{0.157^2} \right) \cdot (0.00349)^2 cms^{-2} \\
&= 12.060cms^{-2}
\end{aligned}
$$

B Questions

- The most likely measurement obtained from a single sample will be the mean of the previous values, because this is the point of highest probability on the gaussian distribution. However, the standard deviation cannot be calculated for a single measurement.

- The most likely result for the mean of the 20 next measurements is the mean of the previous measurements because the mean will converge on the point of highest probability. Assuming that the previous group performed 20 trials, the standard deviation of our measurements will most likely be the same as the standard deviation of the previous set of measurements because the apparatus and measuring procedures are the same and all errors would be expected to remain within the same distribution. However, if we performed more trials then they did, or if we were combining our results with theirs then the standard deviation would be smaller then the original value.

- This result gives the factor of standard deviations away that the constant acceleration calculated from the best results was from the acceleration measured in the worst result.

$$
\begin{aligned}
&= \frac{a_a - a_{10}}{\sigma_{a10}} \\
&= \frac{\left|12.060cms^{-2} - 14.107cms^{-2}\right|}{0.255cms^{-2}} \\
&= 8.03
\end{aligned}
$$

  Because the worst result was off of the measured average by a factor of 8σ, this suggests that there is systematic error in the experiment. Examining Figure 1, it can be seen that the error increases systematically as the time becomes shorter. This error can be explained one of several factors. Possibly, the upper portion of the incline was not perfectly flat, or perhaps the $10cm$ data were recorded with the person stopping the timer sitting at an angle other then perpendicular to the motion, causing parallax errors. The error could also be explained by the cylinder not being perfectly round. Also, a consistent human error in timing reflex could be factored in. Further tests would be needed to determine the exact source of the systematic error. If the error was caused by the location of the observer, repeating the trials again could expose the error. If however, the error was in the equipment, we would have to carefully examine the incline and the cylinder to test their shapes.

- Although the histogram for $N = 20$ doesn't fit well into a gaussian distribution, it can be expected that with greater and greater numbers of measurements, the shape of the distribution would approach a gaussian. Alone, this does not provide sufficient evidence that the distribution is gaussian, examining the other histograms reveals that the variations from a gaussian distribution appear to be random. If they were all added together, the result would approach a gaussian curve, suggesting that higher values of N would cause this to occur as well.