1 Conditional probability

$$P(A|B) = \frac{P(A \cup B)}{P(B)} \tag{1}$$

$$P(B \cap A) = P(B|A)P(A) \tag{2}$$

2 Bayes

$$P(A|B) = P(B|A)\frac{P(A)}{P(B)} \tag{3}$$

3 PDF, CDF, PMF

$f_X(x)$, Probabilty Density F'n;
$F_X(x)$, Cumulative Distribution F'n;
$P_X(x)$, Probability Mass F'n.

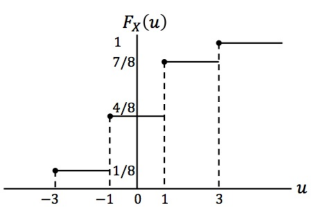
$$f_X(x) = \frac{dF_X(x)}{dx} \tag{4}$$

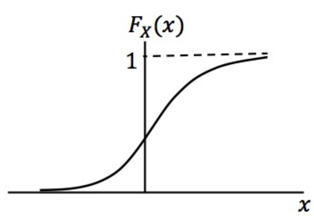
$$F_X(x) = P_X(X \geq x) \tag{5}$$

$$= \int_{-\infty}^{x} f_X(u)\, du \tag{6}$$

$$1 = \int_{-\infty}^{\infty} f_X(u)\, du \tag{7}$$

$$P_X(x) = F_X(x) - F_X(x-) \tag{8}$$





$$F_{X|Y}(x|y) = P_{X|Y}(X \geq x|Y \geq y),\ P(Y = y) > 0 \tag{9}$$

$$= \int_{-\infty}^{x} f_{X|Y}(u|y)\, du \tag{10}$$

$$P_{X|Y}(u|y) = \frac{P_{X,Y}(u, y)}{P_Y(y)},\ Y = y \tag{11}$$

4 Expectation

X random with PMF $P_X(x)$,

$$E[g(X)] = \int_{-\infty}^{\infty} g(X)F_X(x)\, dx \tag{12}$$

$$= \sum_i g(x_i)P_X(x_i) \tag{13}$$

The n^{th} order moment, $E[X^n]$, eg,

$$\mathrm{Mean}(X) = E[X] = \mu_X \tag{14}$$

$$\mathrm{Var}(X) = E[(X - \mu_X)^2] = E[X^2] - {\mu_X}^2 \tag{15}$$

$$\sigma(X) = \sqrt{\mathrm{Var}(X)} \tag{16}$$

$$\mathrm{Cov(X,Y)} = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - \mu_X\mu_Y \tag{17}$$

$$\mathrm{Cov(X,Y)} = 0 \text{ uncorrelated} \tag{18}$$

$$\text{cor coef } \rho(X, Y) = \frac{\mathrm{Cov(X,Y)}}{\sigma_X\sigma_Y} \tag{19}$$

$$\text{cor matr } R_X = E[X \cdot X^T] \tag{20}$$

$$\text{cov matr } C_X = R_X - \mu_X \cdot {\mu_X}^T \tag{21}$$

5 Vectors and Stuff

$$Y = g(X), X, f_X(x) \tag{22}$$

$$f_Y(y) = \sum_i \frac{f_X(x_i)}{\left|\frac{dy}{dx}\right|_{x=x_i}}, g(x_i) = y \tag{23}$$

$$f_Y(y) = f_X(g^{-1}(y))\left|J_{g^{-1}}(y)\right|, \text{ where,} \tag{24}$$

$$\left[J_{g^{-1}}\right]_{ik} = \frac{\partial x_i}{\partial y_k} = \frac{\partial {g_i}^{-1}(y)}{\partial y_k} \tag{25}$$

6 Common

6.1 Uniform (d/c)

$$P_X(x) = \frac{1}{M},\ x \in \{x_1, \ldots, x_M\} \tag{26}$$

$$E[X] = \frac{1}{M}\sum_{i=1}^{M} x_i \tag{27}$$

$$E[X^2] = \frac{1}{M}\sum_{i=1}^{M} x_i^2 \tag{28}$$

$$\mathrm{Var}[X] = E[X] - E[X^2] \tag{29}$$

$$f_X(x) = \frac{1}{b - a}, a \geq x \geq b \tag{30}$$

$$E[X] = \frac{a + b}{2} \tag{31}$$

6.2 Gaussian (c)

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\frac{-(x - \mu)^2}{2\sigma^2}, \sigma > 0 \tag{32}$$

$$E[X] = \mu \tag{33}$$

$$\mathrm{Var}[X] = \sigma^2 \tag{34}$$

6.3 Exponential (c)

Time in between random events; ie $\Gamma(x), \alpha = 1$.

$$f_X(x) = \lambda e^{-\lambda x}, x \geq 0, \lambda > 0 \tag{35}$$

$$E[X] = \lambda^{-1} \tag{36}$$

$$\mathrm{Var}[X] = \lambda^{-2} \tag{37}$$

6.4 Binomial (d)

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x},\ x \in \{0, \ldots, n\} \tag{38}$$

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \tag{39}$$

$$E[X] = np \tag{40}$$

$$\mathrm{Var}[X] = np(1-p) \tag{41}$$

6.5 Bernoulli (d)

Special case of (38,)

$$P_X(x) = p^x (1-p)^{1-x},\ x \in \{0, 1\} \tag{42}$$

$$E[X] = p \tag{43}$$

$$E[X^2] = p \tag{44}$$

$$\mathrm{Var}[X] = p(1-p) \tag{45}$$

6.6 Poisson (d)

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!},\ x \in \{0, 1, \ldots\}, \lambda > 0 \tag{46}$$

$$E[X] = \lambda \tag{47}$$

$$E[X^2] = \lambda(\lambda + 1) \tag{48}$$

$$\mathrm{Var}[X] = \lambda \tag{49}$$

6.7 Geometric (d)

$$P_X(x) = p(1-p)^{x-1},\ x \in \{1, 2, \ldots\},\ 0 > p > 1 \tag{50}$$

$$E[X] = p^{-1} \tag{51}$$

$$\mathrm{Var}[X] = \frac{1-p}{p^2} \tag{52}$$

6.8 Cauchy (c)

$$f_X(x) = \frac{\alpha}{\pi(x^2 + \alpha^2)},\ \alpha > 0 \tag{53}$$

$$E[X^n] = \mathrm{dne} \tag{54}$$

7 MGF

Moment Generating Function (could not exist,)

$$M_X(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx \tag{55}$$

$$E[X^n] = \frac{d^n M_X(t)}{dt^n} \tag{56}$$

8 CF

Characteristic Function (exists,)

$$C_X(t) = E[e^{itX}] = \int_{-\infty}^{\infty} e^{itx} f_X(x)\, dx \tag{57}$$

$$E[X^n] = i^{-n} \left. \frac{d^n C_X(t)}{dt^n} \right|_{t=0} \tag{58}$$

9 Ind't, Stationarity, Wide-Sense Stationarity, and Power Spectrum

Independent, where $a = \{f, F, P\}$,

$$a_{X_1,\ldots,X_N}(x_1, \ldots, x_N) = \prod_{n=1}^{N} a_{X_N}(x_n) \tag{59}$$

stationary,

$$F_{\ldots,X(t_N)}(\ldots, x_N) = F_{\ldots,X(t_N+q)}(\ldots, x_N) \tag{60}$$

WSS,

$$\mu_X(t) = \mu_X \tag{61}$$

$$t_1 - t_2 = \tau R_X(t_1, t_2) = R_X(\tau) \tag{62}$$

PSD (Fourier,)

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-i2\pi f\tau}\, d\tau \tag{63}$$

$$R_X(n) = \int_{-\frac{1}{2}}^{\frac{1}{2}} S_x(f) e^{i2\pi fn}\, df,\ \text{discrete} \tag{64}$$

$$E[X^2(k)] = R_X(0) = \int_{-\frac{1}{2}}^{\frac{1}{2}} S_x(f)\, df \tag{65}$$

$$R_X(\tau) = \frac{E[(X_t - \mu)(X_{t+\tau} - \mu)]}{\sigma^2} \tag{66}$$

10 Misc

The time between events in a Poisson process is an exponential random variable.