"Generalised" Cross-Product

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For all those in MATH-133, here's a great trick that will save you lots and lots of time. I came up with it when I was in MATH-133 several years ago.

Let's start easy; so you've got a vector and you need to get a perpendicular vector.

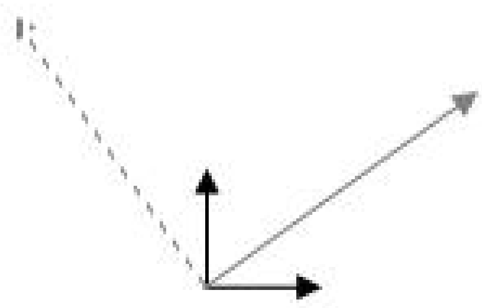

Figure 1: This is the situation.

Well you do it like this:

$$\times(v) = \begin{vmatrix} \hat{x} & \hat{y} \\ v_0 & v_1 \end{vmatrix} \quad (1)$$

$$= v_1\hat{x} - v_0\hat{y} \quad (2)$$

But wait, it generalises:

$$\times(x_0, \dots, x_n) = \begin{vmatrix} \hat{x_0} & \cdots & \hat{x_n} \\ x_{00} & \cdots & x_{0n} \\ \vdots & & \vdots \\ x_{n0} & \cdots & x_{nn} \end{vmatrix} \quad (3)$$

Which the cross-product is a sub-operation of dimension three.

eg

$$\times(a, b, c) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} & \hat{t} \\ a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & b_3 \\ c_0 & c_1 & c_2 & c_3 \end{vmatrix} \quad (4)$$